Journal of Visualization, Vol. 3, No. 4 (2001) 323-330

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Received 27 July 2000.
Revised 28 September 2000.

***Abstract*:** Gravity-driven flow in a soap film tunnel is spatially almost two-dimensional. A modification of particle-image velocimetry technique produces a comprehensive quantitative description of the flow. The technique allows simultaneous acquisition of the velocity (two components) in the plane of the film and the film thickness. The latter behaves as a scalar advected by the flow. The visualization method developed for these data sets uses the thickness field, the vorticity field and the instantaneous velocity streamline pattern for surface elevation, color and bump maps respectively, resulting in color surface images that reveal important features of the flow. In decaying turbulence behind a row of cylinders (2D grid turbulence), the images demonstrate the coarsening of the flow structure with downstream distance, which is the feature specific to turbulence in two dimensions. Strong correlation between thickness fluctuations and vorticity peaks also becomes apparent.

***Keywords*:** soap films, 2D turbulence, PIV.

Soap film flows have attracted considerable recent attention both in the scientific press and in the mass media, including CNN and ABC News. There are several likely reasons for that. The first is that soap films promise an interesting way to study two-dimensional (2D) turbulence, which is important in many atmospheric and oceanic processes. The second reason is that soap film experiments are relatively inexpensive and robust, thus making them attractive for instruction and demonstration purposes. In our perception, however, the main explanation for the popularity of soap film research is its tremendous aesthetic appeal, further augmented by the possibility to see what is happening in the flow either with a naked eye or with some fairly minimal recording equipment.

The most obvious manifestation of soap film is in the walls of a soap bubble. Experimental soap film hydrodynamics usually studies somewhat different objects, but the film is the same − a layer of surfactant solution (1 to 2%) in water sandwiched between two surface layers of the surfactant molecules, with their hydrophobic tails sticking into the air. The thickness of the entire sandwich can vary from fractions of a micron (black films) to tens of microns. Under certain limitations, the equations governing the soap-film flow will be very similar to compressible 2D Navier-Stokes equations, with surface tension taking the role of pressure and local film thickness acting as density (Couder et al., 1989). The early studies had soap films stretched on a stationary frame. Although the concept was originally developed by Sir James Dewar along with the thermos flask and cordite, the honor of reintroducing it in the contemporary scientific framework belongs to Couder (1984). The problem with frames is the sagging and drying of the film, affecting its properties and limiting its lifetime. Soap-film tunnel arrangements, in which the film is constantly replenished, were developed by Gharib and Derango (1989) and by Rutgers et al. (1996). Finally, Vorobieff et al. (1999) developed the tilted gravity-driven tunnel, which combines the simplicity and robustness of the gravity-driven apparatus of Rutgers et al. (1996) with the possibility of studying slow, thick films, similar to those produced by the horizontal tunnel of Gharib and Derango (1989).

As pertains to visualization/diagnostics of soap-film flows, there are several interesting options available. First, as described by Couder (1984), if the film is illuminated by monochromatic light, reflection from the "front" and "rear" surfaces of the film will produce interference fringes due to varicose perturbations in film thickness. The latter behaves like a scalar advected by the flow (not necessarily a passive scalar though), thus making it an attractive quantity to visualize. For really thin films (hundreds of nanometers) illuminated by white light, it is possible to observe beautiful color interference patterns. For example, see Weidman et al. (1997). The interference-fringe visualization is used mostly qualitatively. For quantitative studies, a variety of velocity point-measurement techniques was applied – from laser Doppler velocimetry (Gharib and Derango, 1989; Martin et al., 1998) to optical fiber velocimetry (Rivera et al., 1998). The application of particle-image velocimetry (PIV) to soap-film flows would seem the natural thing to do, because it would combine the quantitative approach with the ability to see the flow structure. In the process of the implementation of PIV for soap films, Rivera et al. (1998) and Vorobieff et al. (1999) found it possible to acquire not only the velocity, but the thickness field as well. Therefore, there is more to soap films than meets the eye – but this wealth of information still needs to be visualized.

This paper deals with the challenge of simultaneous vizualization of quantitative flow- and thickness-fields on the example of a turbulent soap-film flow behind a row of cylinders. Our visualization technique uses film thickness, vorticity and the instantaneous streamlines to produce the elevation, color and texture map for imaging a 2D surface. The resulting images reveal important features of the flow, specifically the coarsening of the flow structures with downstream distance and the strong coupling between vorticity and thickness.

The experimental apparatus is described in detail in our earlier work (Vorobieff et al., 1999). It consists of the bottom and top reservoirs (Fig. 1), the latter connected through a valve to a nozzle supplying the soap solution to two 0.5 mm nylon wires stretched towards the bottom reservoir. The wires are pulled apart by two pairs of hooks, producing a parallel channel 1.2 m long and 6 cm wide. The apparatus is attached to a frame that can be tilted at an arbitrary angle with respect to vertical. The tilting of the soap-film flow reduces the effective component of the gravity force driving the flow. This results in slower and thicker films than those attainable in vertical tunnels (Rutgers et al., 1996). The results presented in this paper were acquired with the tilt angle 82° from vertical and the mean film thickness about 30 μm. Laminary boundary layers form on both "walls" of the tunnel (the nylon wires). In the studies of wake flows, care must be taken to avoid the influence of these boundary layers, thus confining the measurements to the central section of the tunnel, where the freestream velocity field is reasonably uniform.

The acquisition system takes advantage of the flow being essentially confined in 2D. The lighting system employs a pair of xenon flash lamps rather than lasers. The lamps produce short (~5 μs), powerful (up to 0.5 J) pulses of white light (5500K). The flow is seeded with small, bright particles of titanium dioxide. The constraint on the size of the particles is important for soap-film flows. If the particle size is considerably smaller than the mean thickness of the film, one can use the standard procedures to estimate the particle flow-tracking error due to turbulent lag developed for three-dimensional (3D) flows. However, if the particle size is comparable with the film thickness, not only does its flow-tracking fidelity become problematic, but concerns arise whether the particles begin to interfere with the physics of the flow (Fig. 2). An electron microscopy test of the titanium dioxide particles used in our experiments produced a size distribution within the range of 0.2±0.1 μm – much smaller than the mean film thickness of 30 μm, even allowing for the possibility of several particles clumped together. The volume fraction of the particles in the flow is 5×10^{-6}. On average, this amounts to one particle in a cube of film solution 10 μm on the side.

The image acquisition device employed in the experiment was a Pulnix camera with a 1K by 1K CCD array (denoted "PIV" in Fig. 1). For an image area 12 by 12 mm, the resolution is 11.7 μm/pixel. The images were acquired with an f-stop *f*/5.6. At this f-stop, the entire depth (thickness) of the film was imaged sharply. Each frame was double-exposed with a pair of flashes separated by a 200-μs interval, and then analyzed with a standard single-frame cross-correlation PIV algorithm. Estimates of the errors in the PIV velocity-field reconstruction for the experimental apparatus are presented in earlier work (Vorobieff et al., 1999). The main accuracy-limiting factor for these experiments is the pixel resolution, resulting in random error not exceeding ±1.25 cm/s. The quasi-two-dimensional nature of the flow provides another advantage for PIV – there is no out-of-plane velocity component to worry about.

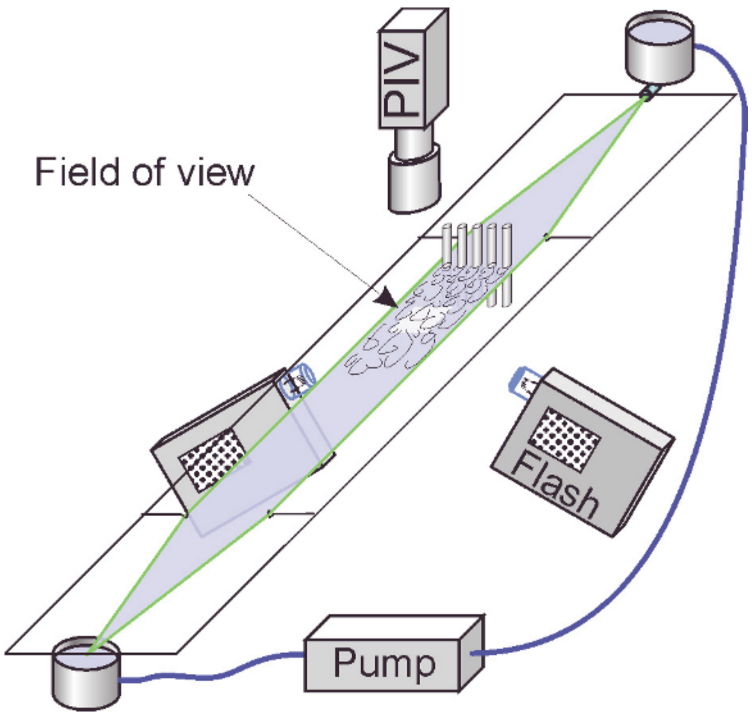

Fig. 1. Schematic of experimental apparatus. Brighter zone in the soap film indicates camera field of view.

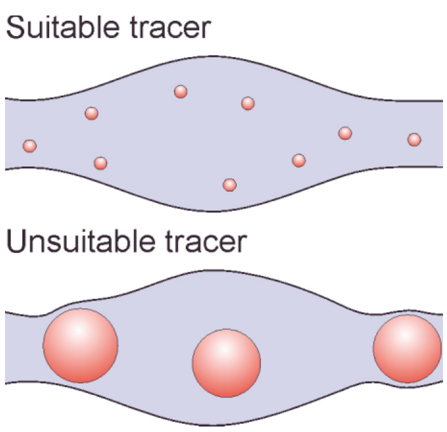

Fig. 2. Tracer particles in soap film.

In addition, the double-exposed images of the particle-seeded film can be analyzed to produce a map of local film thickness (Vorobieff et al., 1999). Thickness information can be recovered by averaging the intensity of light scattered by the seeding particles in the same square window that is used in PIV interrogation, leading to the thickness field quantified on the same grid as the velocity field.

Under the experimental conditions described above, the central section of the tunnel where the velocity is uniform within 10% is about 3 cm wide. The 12 mm square data window was aligned with the axis of the tunnel, leading to freestream velocity nonuniformity in the field of view not worse than 5%.

Perhaps the most interesting and important problem of 2D hydrodynamics is 2D turbulence. Theoretically described in the late sixties by Kraichnan (1967) and Batchelor (1969), 2D Navier-Stokes turbulence should have scalings and behavior significantly different from its 3D counterpart. Consider energy injected into the system at some scale l_0 (corresponding to an injection wave number k_0), leading to formation of a homogeneous turbulent field. In 3D, the energy spectrum $E(k)$ will exhibit $-5/3$ power-law behavior in some range of wave numbers greater than K_0 (the Kolmogorov inertial range). In 2D, the energy cascade with the $-5/3$ power-law spectrum will be inverse – directed towards smaller wave numbers. Along with the inertial range of the inverse energy cascade, in 2D there also will be an enstrophy (mean-square vorticity) inertial range. Within this inertial range confined between the injection scale and the dissipation scale, enstrophy spectra should follow a -1 power law. The first direct confirmation of enstrophy cascade (Rivera et al., 1998) and the simultaneous measurement of power-law behavior in two inertial ranges in soap films (Rutgers, 1998) show strong evidence of consistency between the statistics of soap-film turbulence and the theoretical predictions of Kraichnan (1967) and Batchelor (1969).

Nevertheless, quite a number of unresolved issues remain. One of the most important among them is the coupling between thickness and vorticity fluctuations. Another issue is the influence of the inverse energy cascade on the flow structure – arguably, it should lead to coarsening of structures in 2D turbulence. For a thorough review of these issues and a general overview of soap-film physics, refer to the paper of Couder et al. (1989).

To study soap film turbulence, a spanwise row of cylinders (Fig. 1) is typically inserted into the flow. In earlier work with the tilted soap-film tunnel (Rivera et al., 1998; Vorobieff et al., 1999), instantaneous velocity and thickness measurements were taken downstream of a row of 2.7 mm cylinders separated by 2.7 mm gaps. The corresponding energy injection scale is l_0=2.7 mm and the wavenumber is $k_0=2\pi/l_0$=2.33 mm^{-1}. Multiple data sets were acquired at several downstream distances: immediately behind the cylinders, 3, 5 and 10 cm downstream. The present work continues the analysis of these data sets. An example of the information contained in a single data set is presented in Fig. 3: instantaneous velocity (freestream velocity subtracted in the figure) and thickness fields. In each grid point, there are at least three quantitative measurements to be considered: the velocity components and the thickness. Moreover, the flow can also be quantified in terms of vorticity or local flow topology maps, as well as instantaneous streamline patterns. What will be the most advantageous way to visualize this information? The question is pertinent not just to soap-film research, but to the general problem of 2D hydrodynamics.

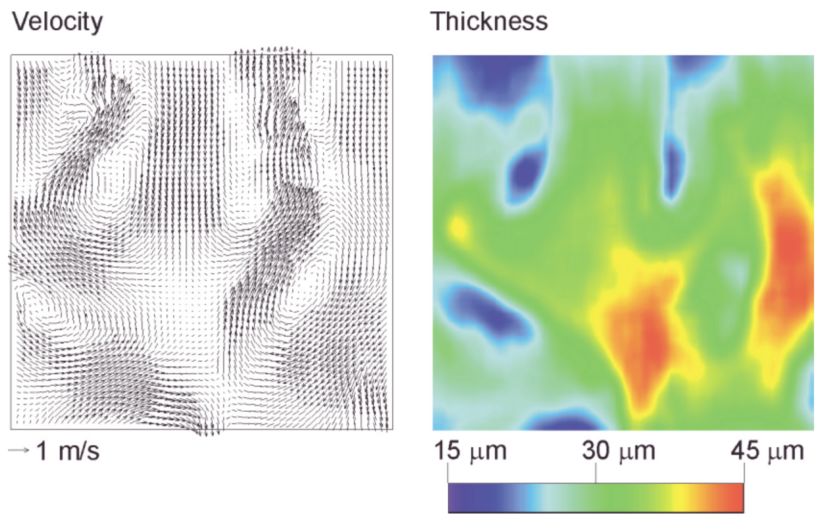

Fig. 3. Instantaneous velocity (freestream subtracted) and thickness field acquired 3 cm downstream of a row of cylinders.

The obvious way to look at the flow is to present the velocity $\boldsymbol{u}=(u,n)$ as a vector field (Fig. 3, left). This plot is not very easy to interpret, however. Visualization of an instantaneous streamline pattern offers an attractive alternative, especially because there is no out-of-plane velocity. Both the velocity vectors and the streamlines, however, are reference-frame dependent and thus can be misleading. The natural reference-frame-independent quantity to image would be the curl of the velocity field (vorticity): $\omega = \nabla \times \boldsymbol{u}$. Also one could look at the invariants of the rate-of-deformation tensor, which are also reference-frame independent and which convey information about the local flow topology. For a 2D velocity field, the rate-of-deformation tensor is

$$\boldsymbol{A} = \begin{bmatrix} \partial u / \partial x & \partial u / \partial y \\ \partial v / \partial x & \partial v / \partial y \end{bmatrix}$$

Its invariants are p=tr $\boldsymbol{A}$ (the velocity field divergence) and q=det $\boldsymbol{A}$. The sign of the characteristic function $f=p^2-4q$ determines the eigenvalues of $\boldsymbol{A}$ and the local character of the flow: real eigenvalues and shear flow if $f>0$, complex-conjugate eigenvalues and swirling flow if $f<0$. If the flow is incompressible, the divergence $p\equiv 0$.

Figure 4 shows the comparison of the instantaneous streamline pattern, the vorticity map, the map of the characteristic function f and the map of the velocity-field divergence p.

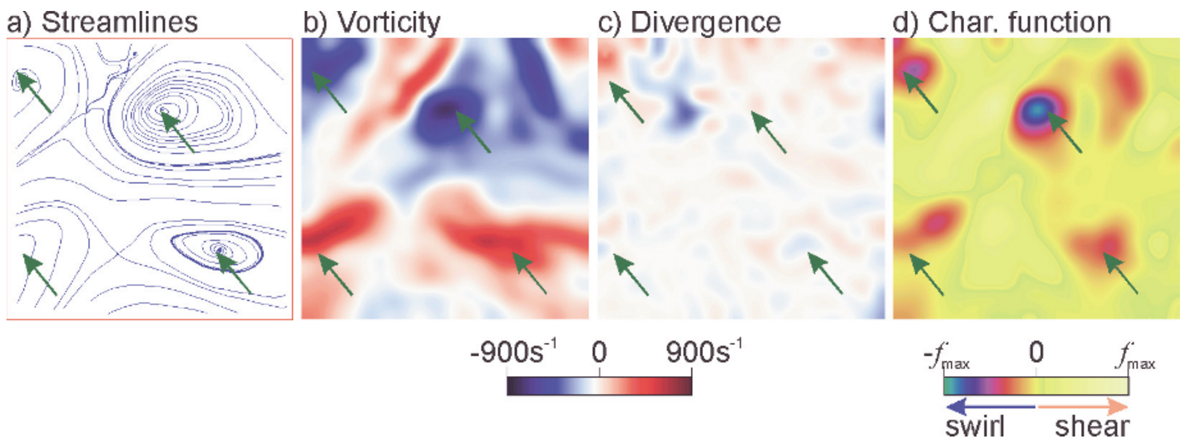

Fig. 4. Instantaneous streamlines (a), vorticity (b), divergence (c), and characteristic function f (d) for the flow field presented in Fig. 3. Green arrows indicate location of vortex cores.

The map of the characteristic function is perhaps the easiest to interpret: zones where the flow is locally dominated by swirl correspond to vortex cores, as becomes apparent when one compares Fig. 4 (d) with either the velocity field in Fig. 3 or the streamlines in Fig. 4 (a). Naturally, the vortex cores also correspond to peaks in the vorticity map (Fig. 4 (b)). However, regions of high vorticity correspond not only to vortex cores, but to zones of high shear in the flow as well. As pertains to the divergence, the map (Fig. 4(c)) is hard to interpret: no features can be easily related to the flow structure, with the possible exception of the region between the two prominent vortices in the upper half of the map, where the negative sign of the divergence indicates a zone of "compression" (thickening of the film). In any case, the mean-square value of the divergence remains on the order of 10% of that of vorticity. Thus the flow is not fully incompressible, but compressibility is not very prominent. It must also be concluded that divergence p, the first invariant of the rate-of-deformation tensor, is not the most interesting quantity to visualize.

Maps of either vorticity or topological characteristic function f present quantitative flow information in an easy-to-interpret form. However, these maps do not describe the thickness field h. Is there a way to present information about both the velocity and the thickness in the same picture? How many quantities can be visualized simultaneously?

Most imaging/rendering tools provide control over several parameters of a generic surface. Besides the shape, the surface can be defined by a color map, bump map, transparency map and so on. Assuming that soap-film data can be presented as a surface plot, the natural quantity to plot as surface elevation is the film thickness: then the shape of the surface will reproduce the shape of the film. A color map can be extracted from a color plot of vorticity or characteristic function f. Finally, the streamline pattern can be employed as the bump map. Other

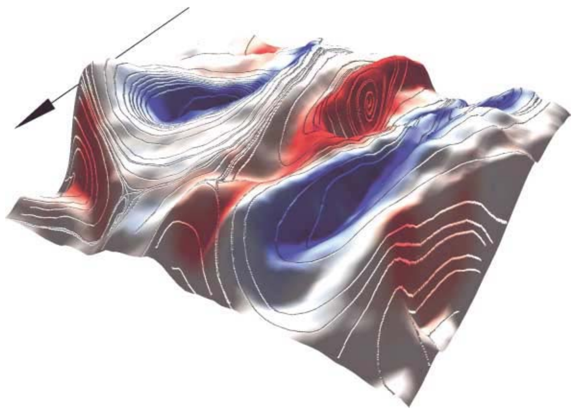

Fig. 5. Instantaneous soap film thickness (elevation), vorticity (color) and streamline pattern (bumps) immediately downstream of the row of cylinders. Vertical scale is 400 times the horizontal scale. The horizontal extent of the window is 12 mm square. The arrow shows the freestream velocity direction.

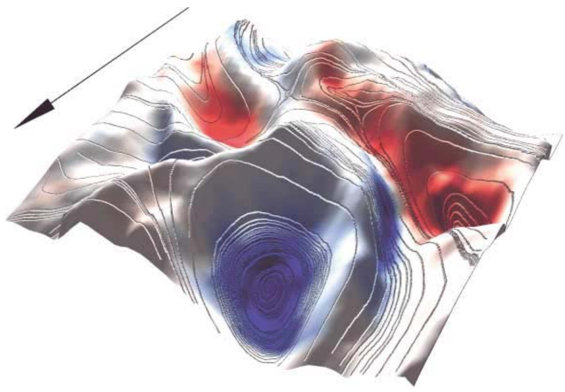

Fig. 6. Instantaneous soap-film flow image 3 cm downstream of the row of cylinders. See caption of Fig. 5 for explanations.

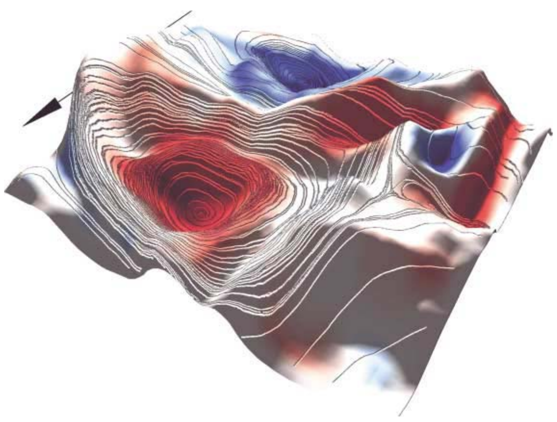

Fig. 7. Instantaneous soap-film flow image 5 cm downstream of the row of cylinders. Refer to caption of Fig. 5 for explanations.

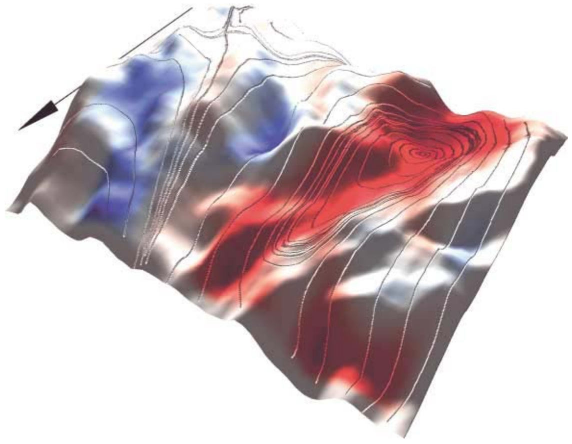

Fig. 8. Instantaneous soap-film flow image 10 cm downstream of the row of cylinders. Refer to caption of Fig. 5 for explanations.

combination for surface imaging are possible (e.g., isocontours of either *f* or); the combination that appears the most elucidating is thickness for surface elevation, vorticity for color and streamline pattern for bumps. Images of the instantaneous flow fields rendered thusly in Autodesk 3D Studio are presented below (Figs. 5 through 8). Vorticity and streamline patterns can be imported into the rendering software as bitmaps, while the surface plot is produced by converting the *x-y-h* gridded data into a DXF surface file and importing the latter. The likely reason that vorticity produces the best results as surface color map is the strong coupling between vorticity and thickness (Vorobieff et al., 1999). Vorticity and thickness fluctuations are born together as the soap film interacts with the cylinders. The behavior of the thickness field is largely consistent with the notion of thickness being passively advected by the flow. The coupling between thickness and vorticity persists as the downstream thickness increases, although the thickness fluctuations decay slower than the vorticity. Thus the most striking feature of all the images is the fact that areas of thinner film correspond to higher vorticity.

The streamline pattern superimposed onto the surface as the bump map suggests that the soap solution is being swept out of the vortex cores. This would imply that thickness does not behave as a "perfect" passive scalar. Indeed, the measurements of the correlation between vorticity and thickness show some increase with downstream distance until the latter exceeds 3 cm. (Vorobieff et al., 1999). Another notable property of the progression of flow images shown in Figs. 5-8 is the coarsening of structures. While there are seven local thickness minima associated with vorticity peaks in Fig. 5, the last figure shows only one vortex spanning the entire extent of the view window. This coarsening of the vortex structure is unique to 2D turbulence.

The PIV-based quantitative measurement technique for flowing soap films produces simultaneous fields of flow velocity and thickness. The most informative way to image these data sets is by fusion of thickness, vorticity and instantaneous streamline data into a 2D surface with a color and bump map. As a side effect, the resulting quantitative images of quasi-2D turbulence in soap films are exceptionally visually appealing, which explains their presence on the pages of Physics Today, 51-10 (1998), 9 and other publications. Visual appeal is the least significant quality of the demonstrated visualization scheme: it reveals important physics of the flow by emphasizing the relationship between film thickness and vorticity and elucidates such important processes as the coarsening of the flow structure. The same visualization scheme can also be applied to a general problem of 2D or quasi-2D flow with scalar advection (the scalar being density, temperature or some other property of interest).

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